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Diffusion and Convection In An Elliptic Tube

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Abstract- This paper presents the theoretical treatment of diffusion and convection in an elliptic tube. And the differential equation for gas or liquid streaming in a tube in steady state in elliptical co-ordinates by using the Marchi-Fasulo transforms technique.

Key Words- Diffusion and convection, Elliptic tube, Marchi-Fasulo transforms

1. INTRODUCTION

Diffusion equation playas an important role in Bilogical science. If we inject a subject in a tube filled with solvent liquid the molecules of the subject are in random motion. They will spread in all directions. Such problem for one dimension when the solvent liquid is not in motion, have been worked out in [1]. Such problems for circular tube, when the diffusion is in direction of flow have been worked out in [1]. Geeta Shrivastava [3] studied the diffusion and convection problem in an elliptical tube .also In this paper, we discuss the same problem by using Marchi-Fasulo transform technique.

2. STATEMENT OF THE PROBLEM

The differential equation for gas or liquid streaming in a tube in steady state in elliptical co-ordinates (z), using McLachlan [2], for diffusion and convection in elliptical tube given by

$$D\left[\frac{2h^{-2}}{\cosh 2\xi-\cos 2\eta}\left(\frac{\partial^2 c}{\partial \xi^2}+\frac{\partial^2 c}{\partial \eta^2}\right)+\frac{\partial^2 c}{\partial \eta^2}\right]-v\frac{\partial c}{\partial z}=0 \qquad (1)$$

Where $x=h\cosh\xi,\cos\eta,$ (2)

$$y=hSinh\xi,\sin\eta, \qquad (3)$$

$$z=z$$

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The direction of flow is supposed to coincide with Z- axis. The boundary conditions are

$$C=C_0 \ at \ Z=0 \quad ; \ 0\le\eta\le 2\pi$$

$$C=0 \ at \ \xi=\xi_0 \quad ; \ 0\le\xi\le\xi_0$$

$$Z>0$$

$$\left[C(\xi,\eta,z)+k_1\frac{\partial C(\xi,\eta,z)}{\partial z}\right]_{z=h}=0 \qquad (4)$$

$$\left[C(\xi,\eta,z)+k_2\frac{\partial C(\xi,\eta,z)}{\partial z}\right]_{z=-h}=0 \qquad (5)$$

Here
C = Concentration of the diffusing substance,
D = Diffusivity,
h = Half the interfocal length of the ellipse,
v= velocity of Liquid,

3. Solution of the problem

Putting C= $R(\xi,\eta).\,Z(z)$ (6)

And applying separation of variable method, we get

$$\frac{2h^{-2}}{(Cosh2\xi-Cos2\eta)}\frac{1}{R}\left(\frac{\partial^2 R}{\partial\xi^2}+\frac{\partial^2 R}{\partial\eta^2}\right)+\frac{1}{Z}\frac{d^2 Z}{dz^2}-\frac{v}{Z}\frac{dZ}{Ddz}=-\lambda^2 \qquad (7)$$

Separating the variables,

$$\frac{1}{Z}\left(\frac{d^2 Z}{dz^2}-\frac{v}{Z}\frac{dZ}{Ddz}\right)=-\lambda^2 \qquad (8)$$

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Or $$\frac{d^2Z}{dz^2}-\frac{vdZ}{Ddz}+\lambda^2 Z=0 \quad (9)$$

$$m=\frac{a\pm\sqrt{a^2-4\lambda^2}}{2}\ where\ a=\frac{v}{D} \quad (10)$$

$$=\frac{v}{2D}\left[1-\frac{\sqrt{v^2-4\lambda^2D^2}}{v}\right] \quad (11)$$

Approximate solution is

$$Z=\exp\left[\frac{v}{2D}\left\{1-\frac{\sqrt{v^2-4\lambda^2D^2}}{v}\right\}z\right] \quad (12)$$

The equation for R is

$$\left(\frac{\partial^2 R}{\partial\xi^2}+\frac{\partial^2 R}{\partial\eta^2}\right)+2q(\cosh 2\xi-\cos 2n)R=0 \quad (13)$$

Taking $4q=h^2\lambda^2$, (14)

The solution when diffusion is symmetrical about both axis of ellipse is give by [2]

$$C=A_{2n,m}Ce_{2n}(\xi,q)ce_{2n}(\eta,q)\exp\left[\frac{v}{2D}\left\{1-\frac{\sqrt{v^2-4\lambda^2D^2}}{v}\right\}z\right] \quad (15)$$

To satisfy the boundary condition C=0 for $\xi=\xi_0$, we must choose q_{2nm} to be the roots of

$Ce_{2n}(\xi,q)=0$ (16)

$$C=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}A_{2n,m}Ce_{2n}(\xi,q_{2n,m})ce_{2n}(\eta,q_{2n,m})\exp\left[\frac{v}{2D}\left\{1-\frac{\sqrt{v^2-4\lambda^2D^2}}{v}\right\}z\right] \quad (17)$$

The coefficients $A_{2n,m}$ are determined by using the boundary conditions Multiply equation(16) on both sides by $(\cosh 2\xi-\cos 2\eta)\,(Ce_{2p}(\xi,q_{2p,r})ce_{2p}(\eta.q_{2p,r})$ and integrate w.r.t. ξ from 0 to ξ_0 and w.r.t.η from 0 to 2π then by the orthogonal property, all terms vanish except p=n , r=m.

Hence

$$A_{2n,m}=\frac{\int_0^{\xi_0}\int_0^{2\pi}Ce_{2n}(\eta,q_{2n,m})c_0(\cosh 2\xi-\cos 2\eta)d\xi.d\eta}{\pi\int_0^{\xi_0}Ce_{2n}^2(\xi,q_{2n,m})[Cosh2\xi-\theta_{2n,m}]d\xi} \quad (18)$$

Where the sum is extended over the positive roots of the equation (17), and

$$\theta_{2n,m}=\frac{1}{\pi}\int_0^{2\pi}Ce_{2n}^2(\eta,q_{2n,m})\cos 2\eta d\eta \quad (19)$$

$$=A_0^{(2n)}A_2^{(2n)}+\sum_{r=0}^{\infty}A_{2r}^{(2n)}A_{2r+2}^{(2n)},] \quad (20)$$

We define the transform with respect to z as

$$\overline{C}(\xi,\eta,\varsigma)=\int_{-h}^{h}C(\xi,\eta,z)P_m(z)dz \quad (21)$$

Its inverse transform is

$$C(\xi,\eta,z)=\sum_{m=1}^{\infty}\frac{\overline{C}(\xi,\eta,\varsigma)}{\lambda_m}P_m(z) \quad (22)$$

Applying the Marchi-Fasulo transform defined in (21) to the equations (1) and using (4), (5) one obtains

$$\left(\frac{2d^{-2}}{\cos(2\xi)-\cos(2\eta)}\right)\left(\frac{d^2\overline{C}}{d\xi^2}+\frac{d^2\overline{C}}{d\eta^2}\right)=a_m{}^2\overline{C} \quad (23)$$

Where the Eigen values a_m are the solutions of the equation

$$[\alpha_1 a\cos(ab)+\beta_1\sin(ab)]\times[\beta_2\cos(ab)+\alpha_2 a\sin(ab)]$$

$$=[\alpha_2 a\cos(ab)-\beta_2\sin(ab)]\times[\beta_1\cos(ab)-\alpha_1 a\sin(ab)]$$

$$\overline{C}(\xi,\eta,\varsigma)=\overline{f}(\eta,\varsigma) \quad (24)$$

Where $\overline{C}$ denotes the Marchi-Fasulo transform of C and ς denotes the Marchi-Fasulo transform parameter.

4. CONCLUSION

In this paper, we have investigated diffusion and convection in an elliptic tube. And differential equation for gas or liquid streaming in a tube in steady state in elliptical co-ordinates by using the Marchi-Fasulo transforms technique. The expressions are represented graphically

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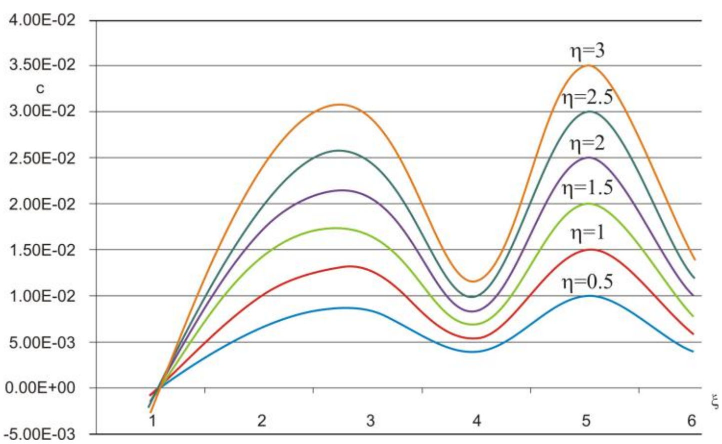
References

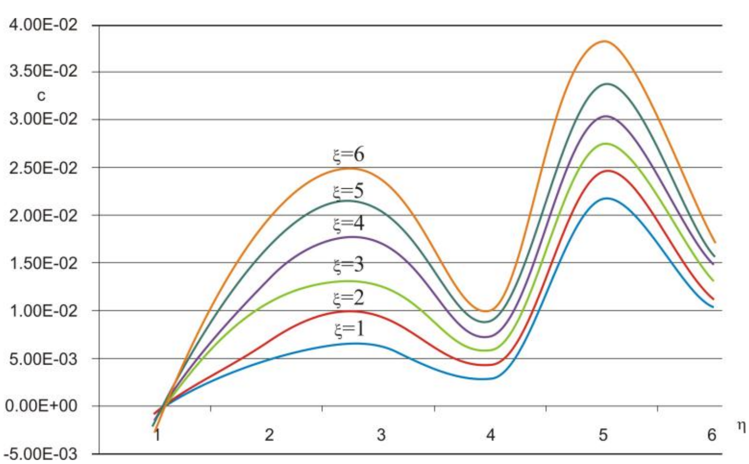
[1]. Jost W (1960) : Diffusion in solids, liquids and gases, Academic Press (1960)

[2]. McLachlan N. W.: Theory and application of Matheu Func tions; Dover Publications (1962)

[3]. Shrivastava Geeta: Ph.D. Thesis, Ch. 2 (2003) submitted to R.D.V.V., Jabalpur.



Graph C Versus ξ For different value η



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